TIME SERIES ANALYSIS FOR ENERGY DATA

M7 - Introduction to Forecasting

Prof. Luana Medeiros Marangon Lima, Ph.D.

Nicholas School of the Environment - Duke University

Learning Goals

- Intro to Forecasting
- Simple Averaging techniques
- Forecasting with ARIMA models
- Intro to Forecasting in R

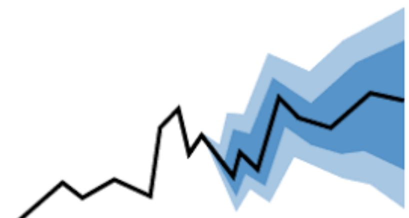
Intro to Forecasting

Intro to Time Series Forecasting

- Assume that future values of the time-series can be estimated from past values of the time-series
- Simple Forecasting techniques
  - Naïve Forecast
  - Simple Average
  - Moving average
  - Weighted moving average
  - Exponential smoothing

Introduction to Forecasting

- **Forecast** → **statement about the future value** of a **variable of interest**
  - Forecasts are often used for weather, demand, and resource availability
  - Important element in making informed decisions

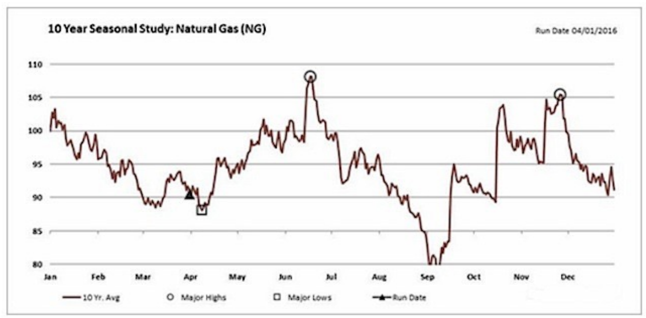


Forecasts affect decisions

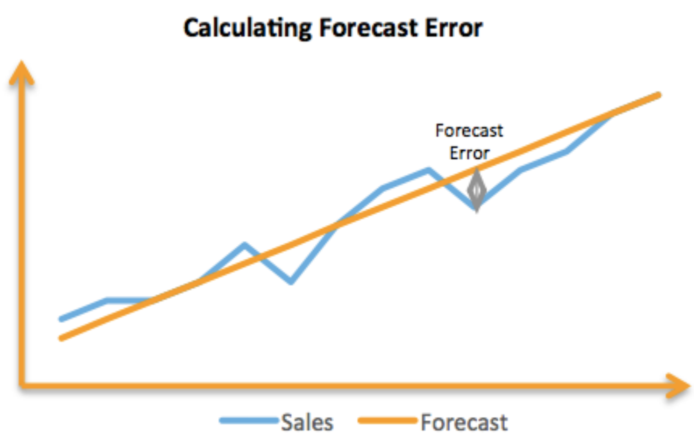
Accounting	Cost/profit estimates
Finance	Cash flow and funding
Human Resources	Hiring/recruiting/training
Marketing	Pricing, promotion, strategy
Operations	Schedules, workloads
Product/service design	New products and services

Two Important Aspects of Forecasts

- Expected level of demand
  - The level of demand may be a function of some structural variation such as trend or seasonal variation



- Accuracy
  - Related to the potential size of forecast error



Features Common to All Forecasts

1. Techniques **assume some underlying causal system that existed in the past will persist** into the future
2. **Forecasts are not perfect**
3. **Forecasts for groups of items are more accurate** than those for individual items
4. **Forecast accuracy decreases** as the forecasting **horizon increases**

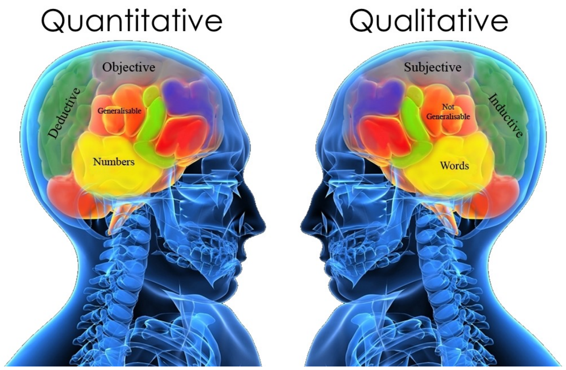
Elements of a Good Forecast

The forecast:

- Should be ***timely***
- Should be ***accurate***
- Should be ***reliable***
- Should be **expressed in *meaningful units***
- Technique should be *simple to understand and use*
- Should be ***cost effective***

Forecasting Process Steps

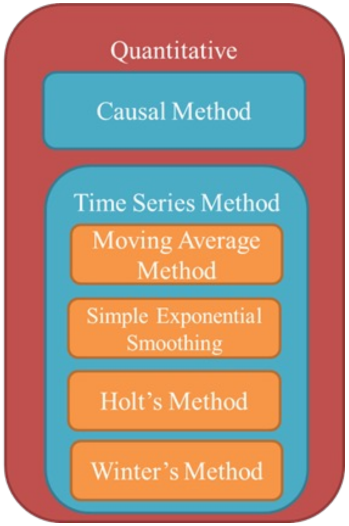
1. Determine the purpose of the forecast
2. Establish a time horizon
3. Obtain, clean, and analyze appropriate data
4. Select a forecasting technique
5. Make the forecast
6. Monitor the forecast



Forecasting Approaches

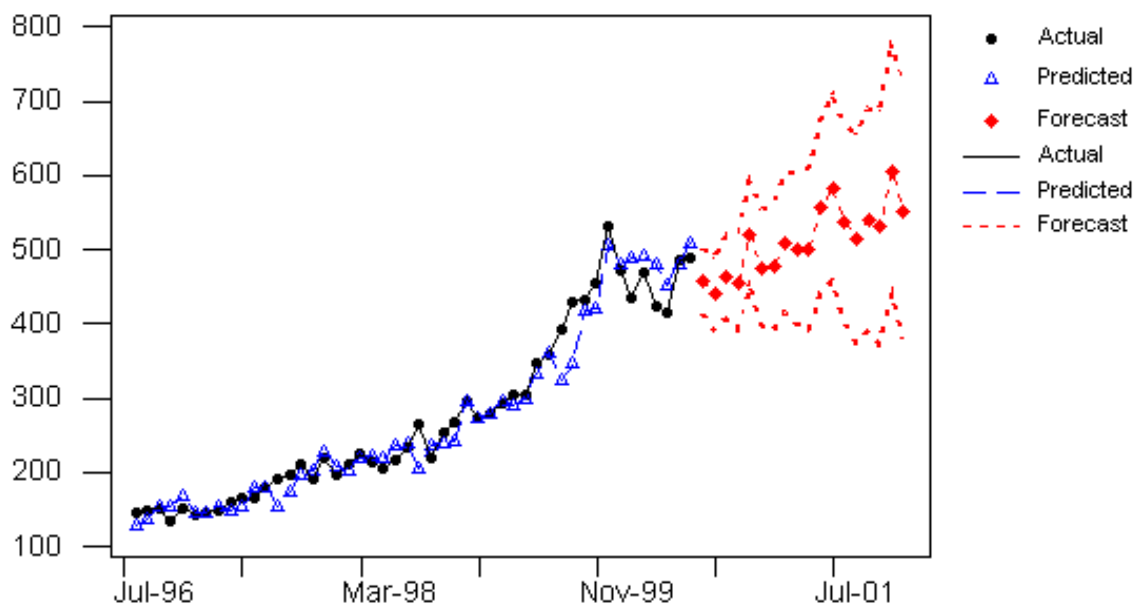
- **Qualitative Forecasting**
  - Qualitative techniques permit the inclusion of *soft* information such as:
    - Human factors
    - Personal opinions
    - Hunches
  - These factors are difficult, or impossible, to quantify

- **Quantitative Forecasting**
  - Involve either the projection of historical data or the development of associative methods that attempt to use *causal variables*
  - These techniques rely on *hard* data



Quantitative Forecasting

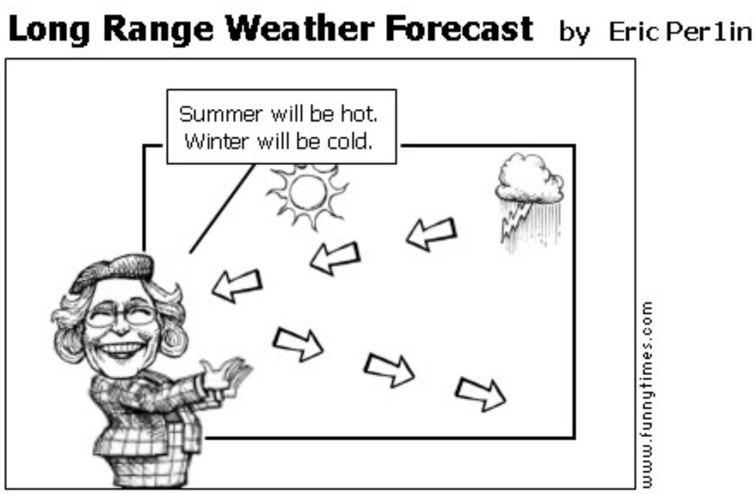
- Forecasts that project patterns identified in recent time-series observations
- Assume that future values of the time-series can be estimated from past values of the time-series



Simple Averaging Forecasting Methods

Time Series Forecasting - Naïve Forecast

- Naïve Forecast
  - Uses a **single previous value** of a time series as the basis for a forecast
  - The forecast for a time period is equal to the previous time period value
  - Can be used with
    - a stable time series
    - seasonal variations
    - trend



Naïve Forecasts

- Forecast for any period = previous period's actual value

$$F_t = A_{t-1}$$

F: forecast   A: Actual   t: time period

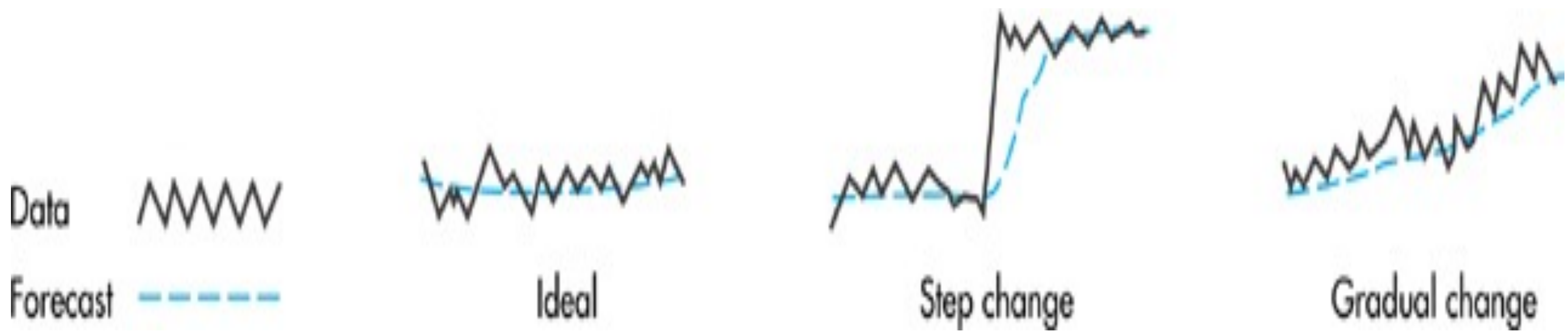
Naïve Forecast Example

Week	Sales (actual)	Sales (forecast)	Error
t	A	F	A - F
1	20	-	
2	25	20	5
3	15	25	-10
4	30	15	15
5	27	30	-3

Naïve Forecasts

- Simple to use
- Virtually no cost
- Quick and easy to prepare
- Data analysis is nonexistent
- Easily understandable
- Cannot provide high accuracy

Uses for Naïve Forecasts



Time Series Forecasting - Averaging

- These techniques work best when a series tends to vary about an average
- Averaging techniques **smooth variations** in the data
- They can handle **step changes or gradual changes** in the level of a series
- Techniques
  1. Moving average
  2. Weighted moving average
  3. Exponential smoothing

Moving Average

- Technique that averages a number of the most recent actual values in generating a forecast

$$F_t = \mathrm{MA}_n = \frac{\sum_{i=1}^{n} A_{t-i}}{n}$$

where

$$F_t = \text{Forecast for time period } t$$

$$\mathrm{MA}_n = n \text{ period moving average}$$

$$A_{t-1} = \text{Actual value in period } t-1$$

$$n = \text{Number of periods in the moving average}$$

Moving Average

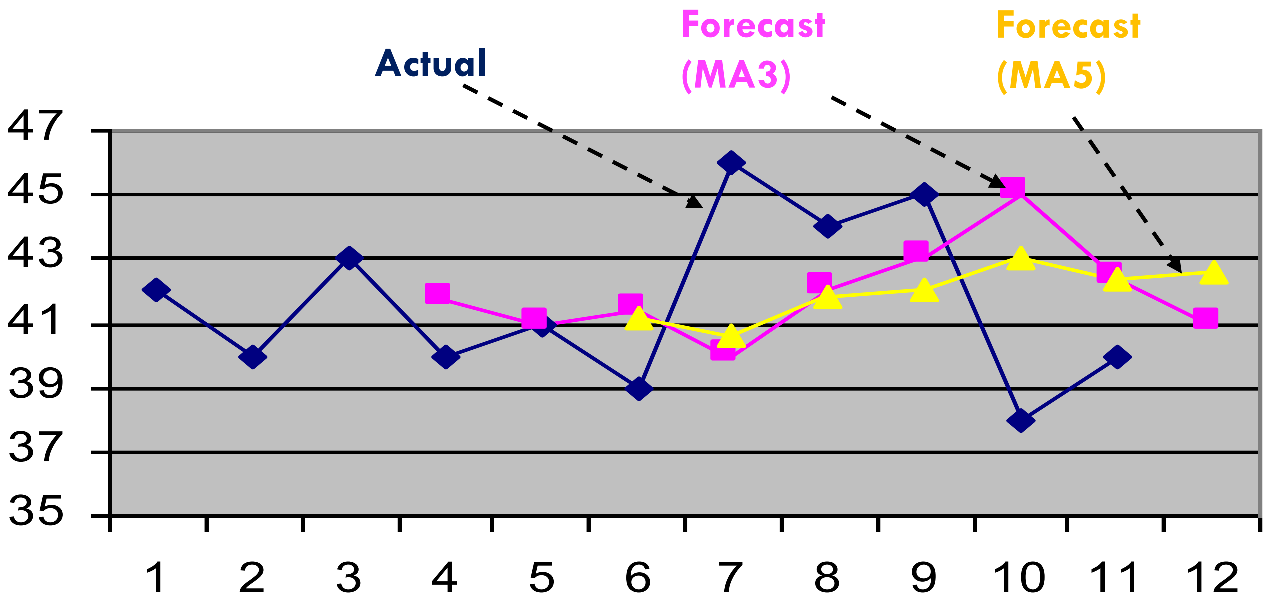
- As **new data become available**, the **forecast is updated** by adding the newest value and dropping the oldest and then re-computing the average

- The number of data points included in the average determines the model's sensitivity
  - **Fewer data points used-- more responsive**
  - **More data points used-- less responsive**

Moving Average Example

Week	Sales (actual)	Sales (forecast)	Error
t	A	F = MA3	A - F
1	20	-	
2	25	-	
3	15	-	
4	30	20	10
5	27	23.3333	3.66667
6		24	

Simple Moving Average



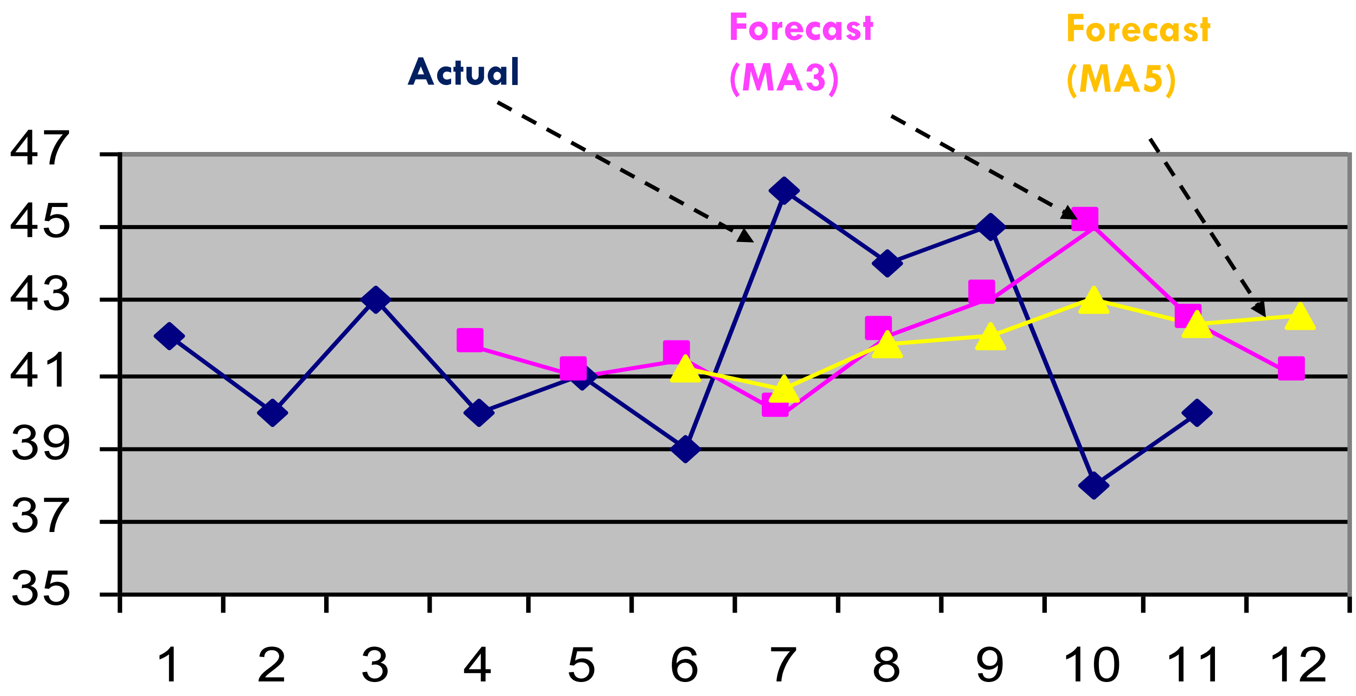
Questions:

- Why is MA3 longer than MA5?
- Which curve fluctuate the most?
- Which curve is the smoothest?

Simple Moving Average

Responsiveness vs. Stability

- Smaller m, responsiveness ↑, stability ↓
- Larger m, responsiveness ↓, stability ↑
- Must maintain stability when fluctuations are high



Weighted Moving Average

- The most recent values in a time series are given more weight in computing a forecast
  - The choice of weights, w, is somewhat arbitrary and involves some trial and error

$$F_{t+1} = w_t A_t + w_{t-1} A_{t-1} + w_{t-2} A_{t-2} + \cdots + w_{t-n} A_{t-n}$$

where

w_t = weight for period t, w_{t-1} = weight for period $t-1$, etc.

A_t = the actual value for period t, A_{t-1} = the actual value for period $t-1$, etc.

Weighted Moving Average Example

Week	Sales (actual)	Sales (forecast)	Error
t	A	F = MA3	A - F
1	20	-	
2	25	-	
3	15	-	
4	30	19	11
5	27	24.5	2.5
6		25.5	

$w_{t-1} = 0.5,$ $w_{t-2} = 0.3,$ $w_{t-3} = 0.2,$

Exponential Smoothing

- A weighted averaging method that is based on the previous forecast plus a percentage of the forecast error

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

F_t: *forecast for period t*

F_{t-1}: *forecast for previous period t-1*

α: *smoothing constant*

A_{t-1}: *actual value from previous period*

$A_{t-1} - F_{t-1}$ *is the error term*

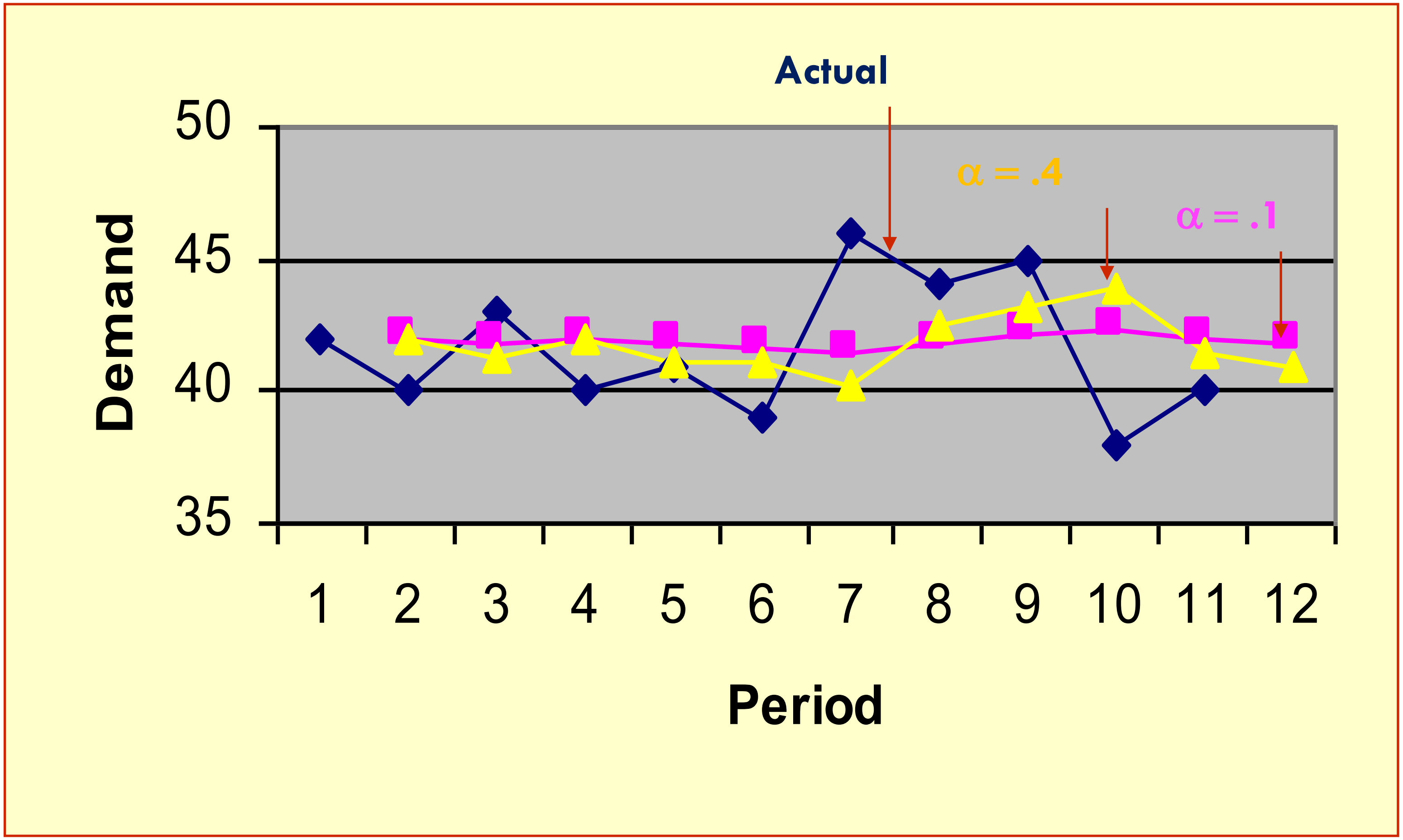
Example 3 - Exponential Smoothing

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Period	Actual	Alpha = 0.1	Error	Alpha = 0.4	Error
1	42				
2	40	42	-2.00	42	-2
3	43	41.8	1.20	41.2	1.8
4	40	41.92	-1.92	41.92	-1.92
5	41	41.73	-0.73	41.15	-0.15
6	39	41.66	-2.66	41.09	-2.09
7	46	41.39	4.61	40.25	5.75
8	44	41.85	2.15	42.55	1.45
9	45	42.07	2.93	43.13	1.87
10	38	42.36	-4.36	43.88	-5.88
11	40	41.92	-1.92	41.53	-1.53
12		41.73		40.92	

$$\boxed{F_3 = 42 + 0.1(40 - 42) = 41.8}$$

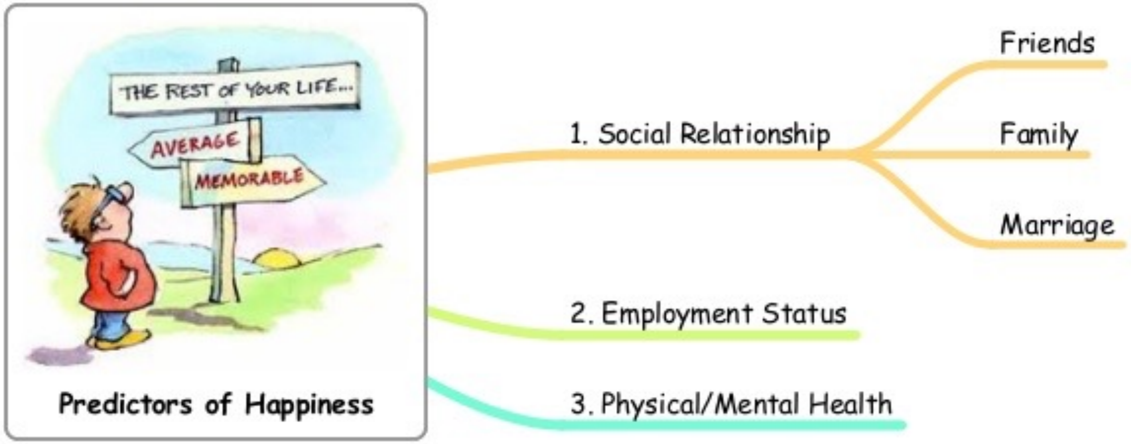
Picking a Smoothing Constant



The smaller alpha represents the smoother series.

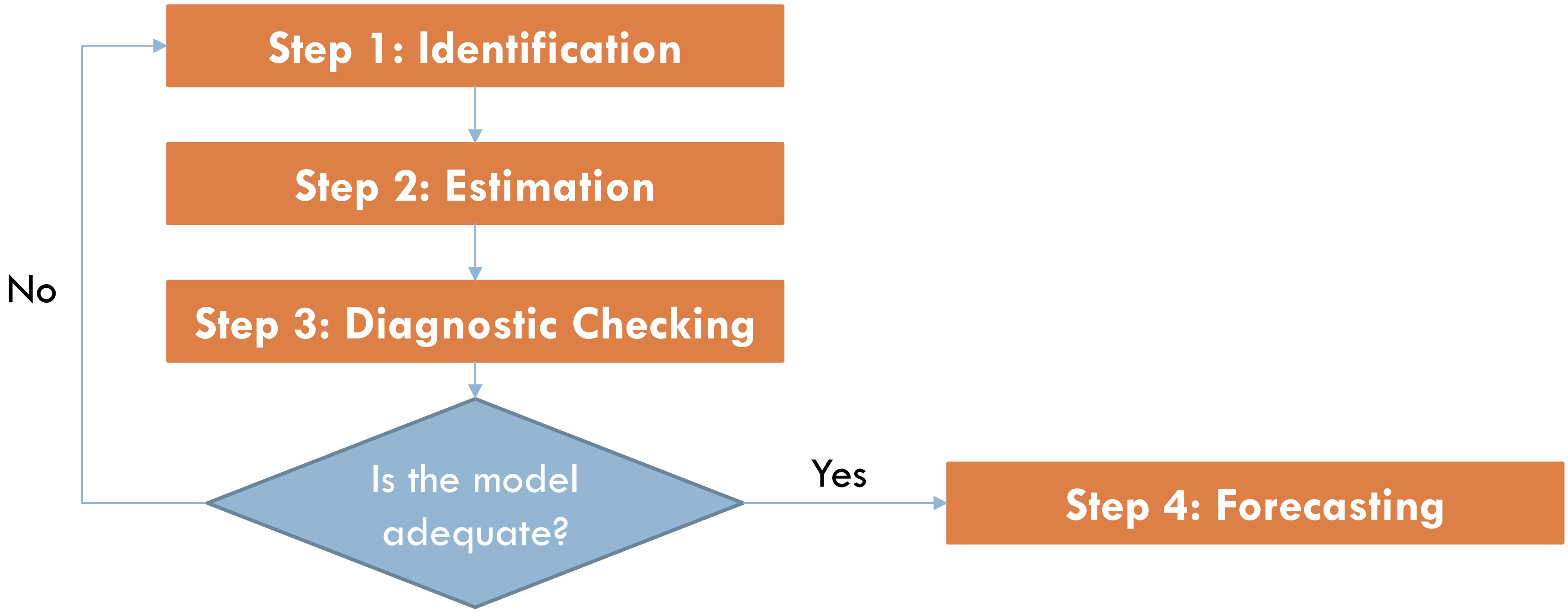
Associative Forecasting Techniques

- Associative techniques are **based on the development of an equation that summarizes the effects of predictor variables**
  - **Predictor variables** - variables that can be used to predict values of the variable of interest
  - **Home values may be related** to: home and **property size, location, number of bedrooms**, and number of bathrooms



Forecasting with ARIMA Models

ARIMA Modeling Process



- The use of ARIMA is appropriate when
  - little or nothing is known about the dependent variable being forecasted,
  - the independent variables known to be important cannot be forecasted effectively
  - all that is needed is a one or two-period forecast

ARIMA Forecasting

- Recall the ARMA(1,1) model equation

$$Y_t = \phi_1 Y_{t-1} + a_t - \theta_1 a_{t-1} \qquad for\ t = 1,2,\ldots,n$$

$$a_t \sim i.i.d.\ N(0, \sigma^2)$$

- What would be the value of the next Y, say $Y_{t+1} = ?$
- From the estimation step you have $\phi = (\phi_1\ \phi_2\ \ldots \phi_p)'$ and σ^2
- All you need to do is plug in the parameters and past Y values
- Same principle is extended for the more general class of ARIMA Models

Forecasting with MA(2)

- MA(2) model $$Y_t = \mu + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$
- Forecasting
  - 1-step

$$Y_{n+1} = \mu + a_{n+1} - \theta_1 a_n - \theta_2 a_{n-1}$$
$$\hat{Y}_{n+1} = \mu + \qquad -\theta_1 a_n - \theta_2 a_{n-1}$$

$E[a_{n+1}] = 0$

$$Y_{n+1} - \hat{Y}_{n+1} = a_{n+1} \Rightarrow Var\left(Y_{n+1} - \hat{Y}_{n+1}\right) = Var(a_{n+1}) = \sigma^2$$

  - 2-step

$$Y_{n+2} = \mu + a_{n+2} - \theta_1 a_{n+1} - \theta_2 a_n$$
$$\hat{Y}_{n+2} = \mu \qquad - \theta_2 a_n$$

$E[a_{n+1}] = 0$
$E[a_{n+2}] = 0$

$$Y_{n+2} - \hat{Y}_{n+2} = a_{n+2} - \theta_1 a_{n+1} \Rightarrow Var\left(Y_{n+2} - \hat{Y}_{n+2}\right) = \sigma^2(1 + \theta_1^2)$$

  - 3-step

$$Y_{n+3} = \mu + a_{n+3} - \theta_1 a_{n+2} - \theta_2 a_{n+1}$$
$$\hat{Y}_{n+3} = \mu$$

$E[a_{n+1}] = 0$
$E[a_{n+2}] = 0$
$E[a_{n+3}] = 0$

$$Y_{n+3} - \hat{Y}_{n+3} = a_{n+3} - \theta_1 a_{n+2} - \theta_2 a_{n+1} \Rightarrow Var\left(Y_{n+3} - \hat{Y}_{n+3}\right) = \sigma^2(1 + \theta_1^2 + \theta_2^2)$$

Forecasting with AR(2)

- AR(2) model $$Y_t = \delta + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + a_t$$
- Forecasting
  - 1-step

$$Y_{n+1} = \delta + \phi_1 Y_n + \phi_2 Y_{n-1} + a_{n+1}$$
$$\hat{Y}_{n+1} = \delta + \phi_1 Y_n + \phi_2 Y_{n-1}$$

$E[a_{n+1}] = 0$

$$Y_{n+1} - \hat{Y}_{n+1} = a_{n+1} \quad \Longrightarrow \quad Var\left(Y_{n+1} - \hat{Y}_{n+1}\right) = Var(a_{n+1}) = \sigma^2$$

  - 2-step

$$Y_{n+2} = \delta + \phi_1 Y_{n+1} + \phi_2 Y_n + a_{n+2}$$
$$\hat{Y}_{n+2} = \delta + \phi_1 \hat{Y}_{n+1} + \phi_2 Y_n$$

$E[a_{n+2}] = 0$

$$Y_{n+2} - \hat{Y}_{n+2} = a_{n+2} + \phi_1 a_{n+1} \quad \Longrightarrow \quad Var\left(Y_{n+2} - \hat{Y}_{n+2}\right) = \sigma^2(1 + \phi_1^2)$$

  - 3-step

$$Y_{n+3} = \delta + \phi_1 Y_{n+2} + \phi_2 Y_{n+1} + a_{n+3}$$
$$\hat{Y}_{n+3} = \delta + \phi_1 \hat{Y}_{n+2} + \phi_2 \hat{Y}_{n+1}$$

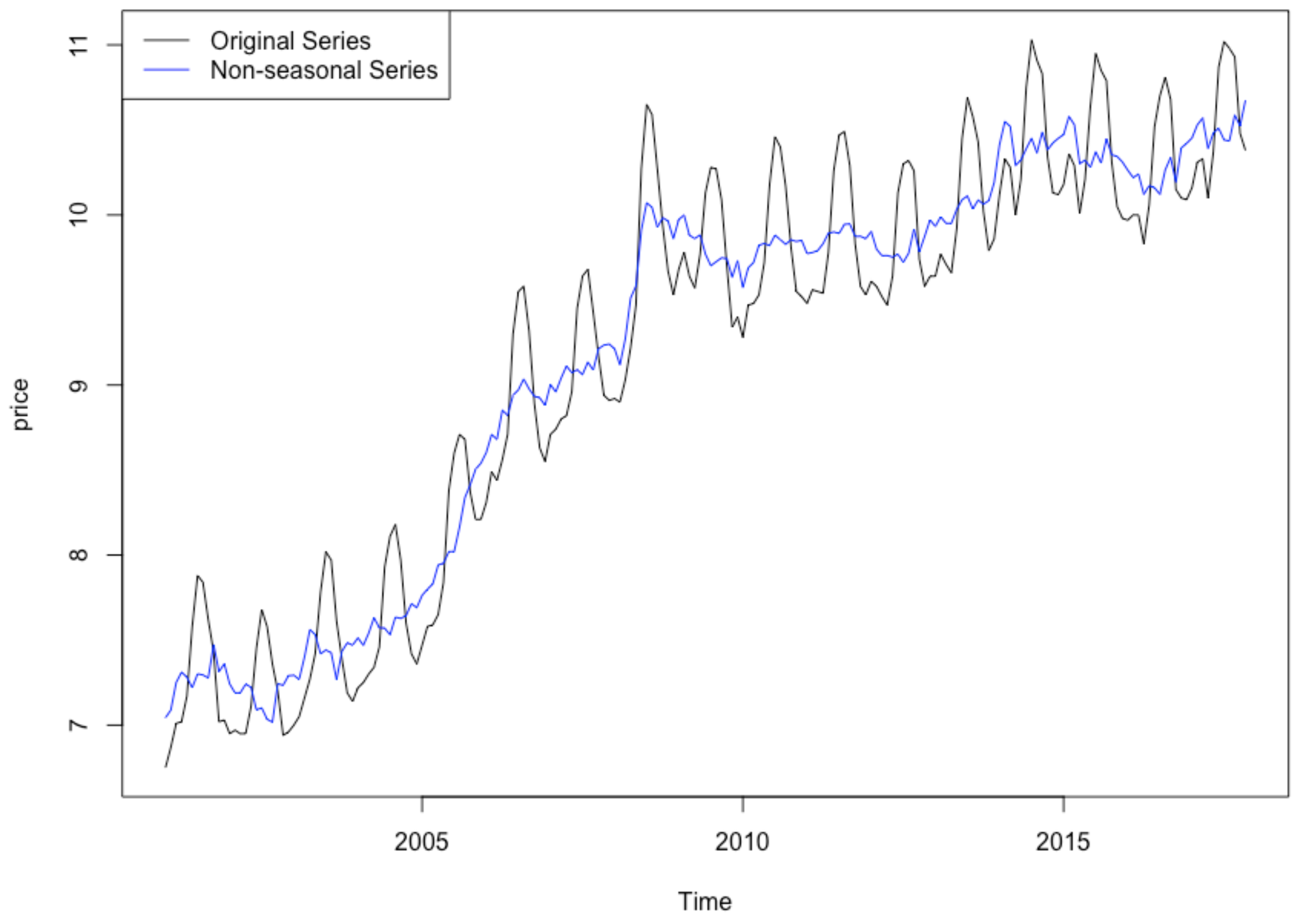
$E[a_{n+3}] = 0$

$$Y_{n+3} - \hat{Y}_{n+3} = a_{n+3} + \phi_1 a_{n+2} + (\phi_1^2 + \phi_2) a_{n+1} \quad \Longrightarrow \quad Var\left(Y_{n+3} - \hat{Y}_{n+3}\right) = \sigma^2(1 + \phi_1^2 + (\phi_1^2 + \phi_2)^2)$$

Let's revisit the electricity price data example…

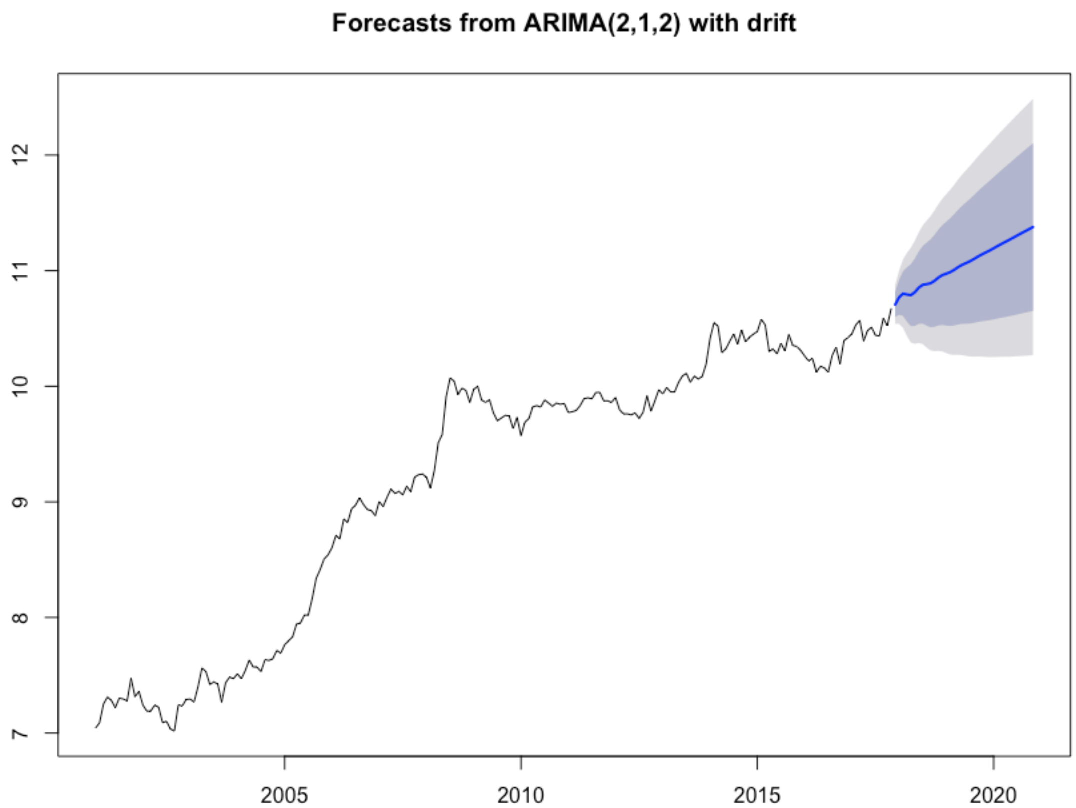
From previous analysis

- Data exhibits trend and seasonality



Example for Energy Price Data

- Taking seasonality out before fitting the ARIMA model



Forecast estimates are provided with confidence bounds

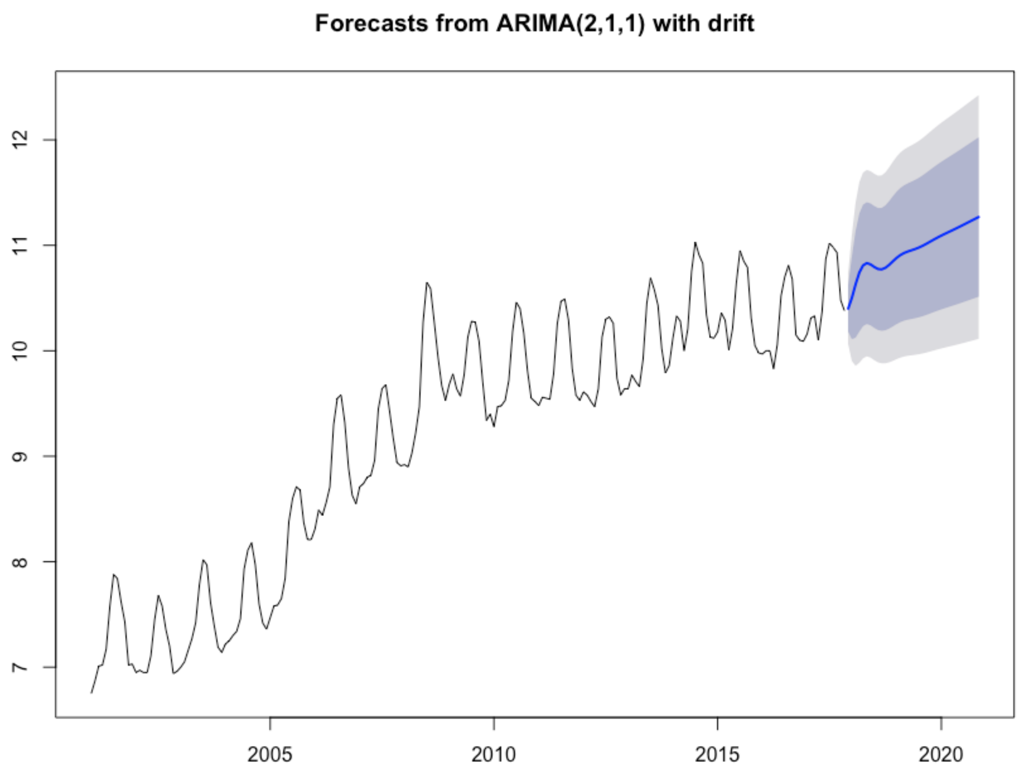
80% in darker blue, and
95% in lighter blue

Longer term forecasts will have more uncertainty, as the model will regress future Y on previously predicted values

As a result the shape of the confidence bounds start to widen as we increase horizon

Example for Energy Price Data

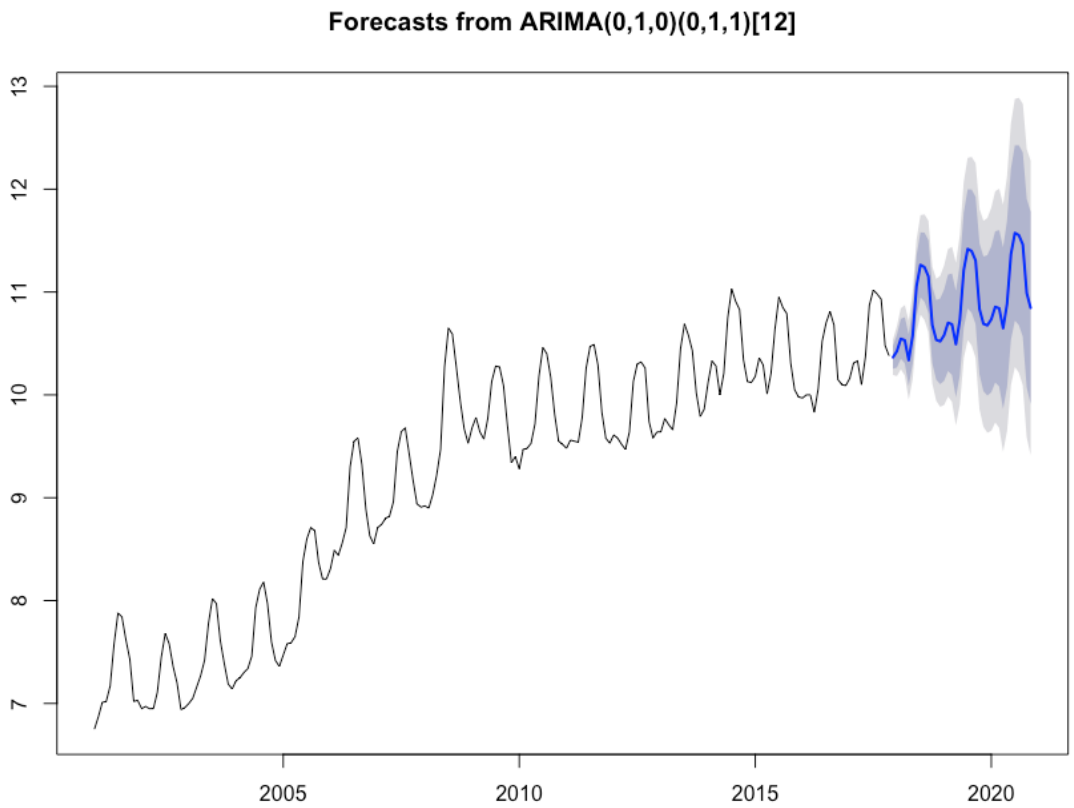
- Note what happens if you don't take seasonality and run the non-seasonal ARIMA



The results will be similar to the naïve forecasting, you are just modeling the trend, but there is no seasonal variation on the forecasts.

Example for Energy Price Data

- Now consider the SARIMA on original data



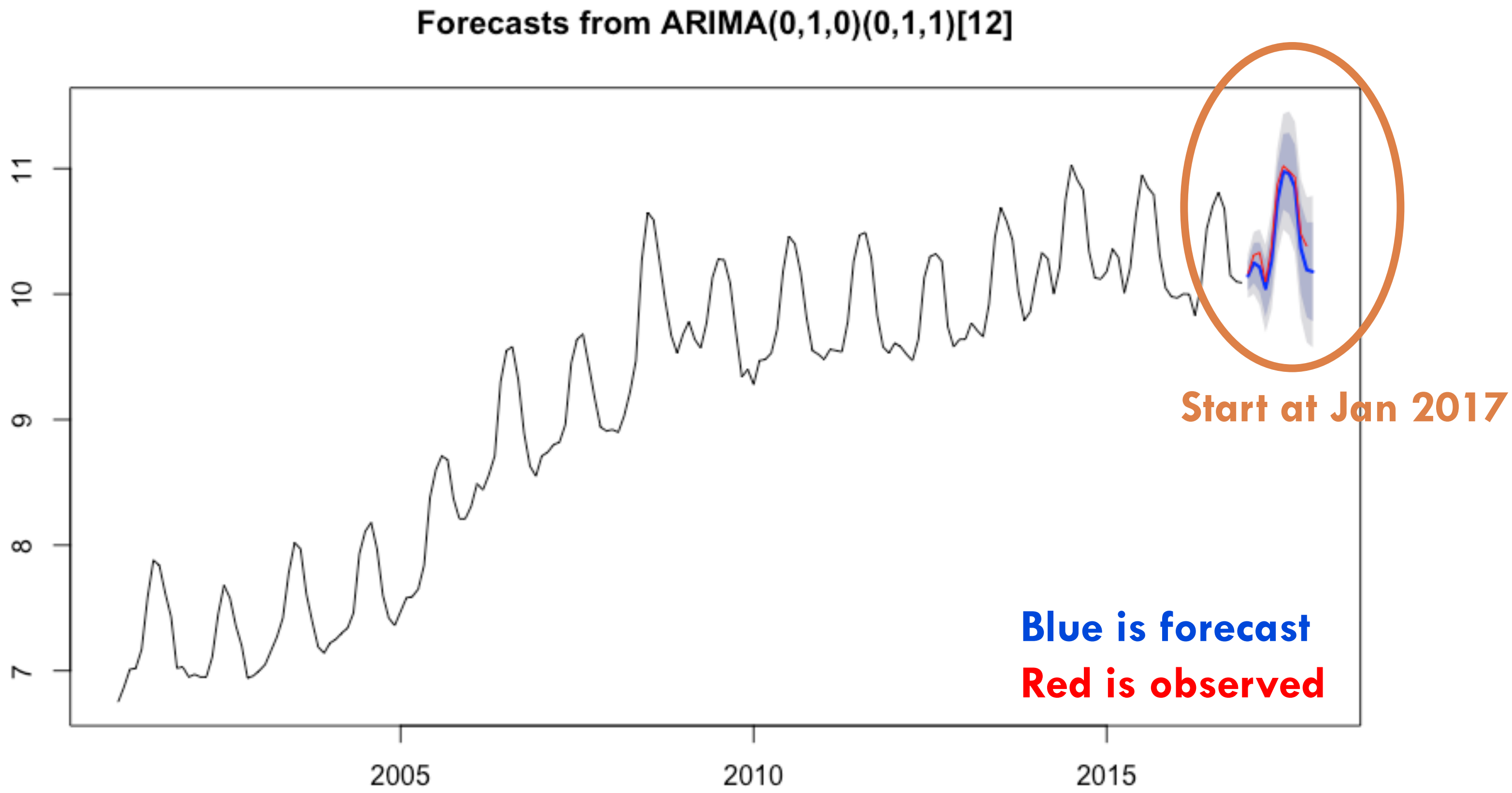
Note that now the forecasts reproduce both trend and seasonal component

How do you check model performance?

- Suppose you want to compute forecast error
- Forecasts can be either in-sample or out-of-sample
- In general the out-of-sample forecasts are a better test of how well the model works, as the forecast uses data not included in the estimation of the model
  - If you want to get a sense of how the model will perform in the future, reserve a portion of the data as a "hold-out" set, fit the model, and then compare the forecast to the actual observed values

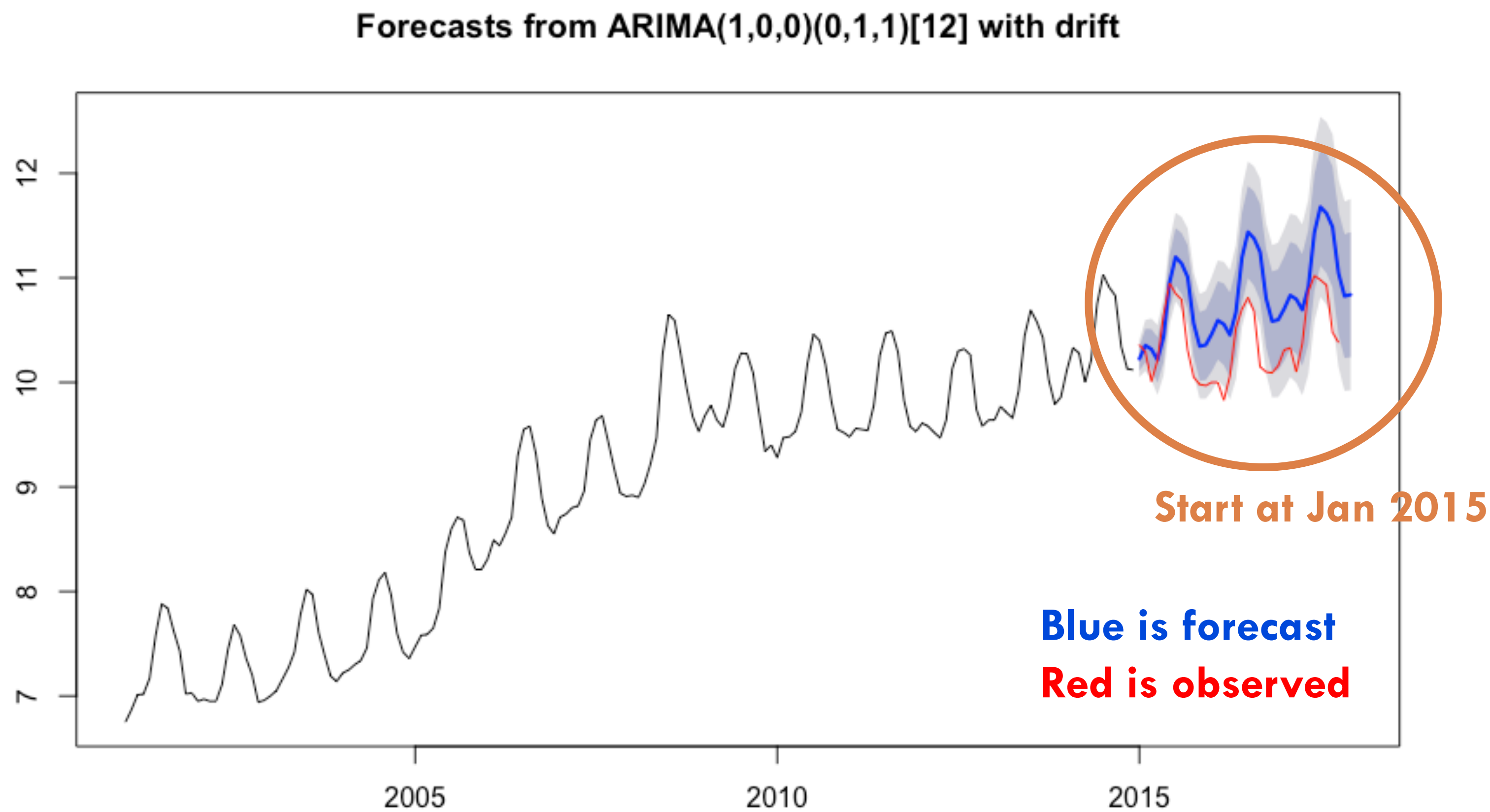
ARIMA Model Performance – Short-term

- To check your model performance over a year, leave one year of data out of the analysis



ARIMA Model Performance – Long-term

- To check your model performance over three years, leave three years of data out of the analysis



Let’s see this in R

Forecasting in R

package "forecast"

package "smooth"

- Forecasting with ARIMA model
  1. *Fit ARIMA using* ***auto.arima() or Arima()***
  2. ***forecast(object, ...)***

- Forecasting with Moving Average Model

  sma(y, order = NULL, h = 10, holdout = FALSE, level = 0.95, silent = c("all", "graph", "legend", "output", "none"), ...)

- Forecasting with Exponential Smoothing

  ses(y, h = 10, level = c(80, 95), alpha = NULL, ...)

More Simple Forecasting Methods in R

package "forecast"

- Arithmetic Mean

  meanf(y, h = 10, level = c(80, 95),…)

- Naïve and Seasonal Naïve Method

  naive(y, h = 10, level = c(80, 95), …)

  snaive(y, h = 10, level = c(80, 95), …)*

  *need to specify frequency when defining ts() object

More Simple Forecasting Methods in R

- All these functions output a forecast object similar to the ***forecast() function***
- The forecast object contains

Original series

Prediction Interval

Residuals and in-sample one-step forecasts

Point forecasts

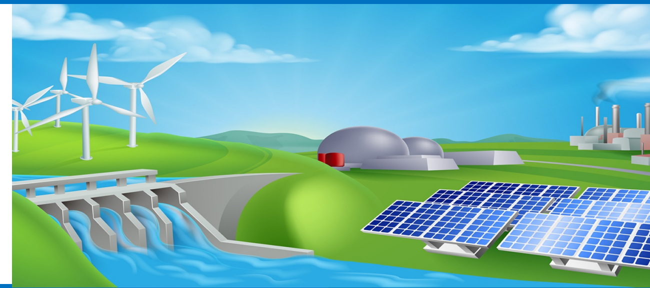
Forecasting Method

Measuring Forecast Accuracy in R

accuracy(f, x, test = NULL, d = NULL, D = NULL, ...)

package "forecast"

- *f* needs to be an object of class "forecast"
- *x* numerical vector containing actual values of the same length as object
- The measures calculated are:
  - ME: Mean Error
  - RMSE: Root Mean Squared Error → MSE
  - **MAE: Mean Absolute Error** → MAD
  - MPE: Mean Percentage Error
  - **MAPE: Mean Absolute Percentage Error**
  - MASE: Mean Absolute Scaled Error
  - ACF1: Autocorrelation of errors at lag 1.




THANK YOU !

luana.marangon.lima@duke.edu